Pricing distortions in multi-sided platforms[*]

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Abstract

We consider the reasons why a monopoly multi-sided platform may price differently from a social planner. The existing literature has focused only on the classical market power distortion and a distortion in the spirit of Spence. We show two additional distortions appear in the presence of cross-group network effects, which we call the displacement distortion and the scale distortion. We show conditions under which the displacement distortion exactly offsets the Spence distortion, and provide an example in which the total of these different distortions results in monopoly prices per user that are lower than the social planner's on both sides. Our results have implications for regulatory policy, which we briefly discuss.

1 Introduction

The striking success of large multi-sided platforms like those run by Alibaba, Amazon, Apple, Facebook, Google, Microsoft, and Tencent is no doubt driven in good part by the positive network effects these platforms enjoy. But this has also generated intense regulatory interest, which has mainly focused on the potential abuse of their market power via practices such as restrictive platform access, bundling to leverage market power, exclusive deals, self preferencing, and price parity clauses. Various policymakers and commentators have also called out the high prices charged to the seller-side of some of these multi-sided platforms (e.g. appstores, credit card platforms, Facebook and Google's ads, and hotel booking platforms).

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In this paper we take a step back and ask a more basic question—what are the pricing distortions implied by a monopoly two-sided platform that enjoys network effects. The common understanding is that aside from a classical market-power distortion in which firms restrict output to increase price above cost, there is an additional distortion due to profit maximization, which has been referred to as the Spence distortion (Katz, 2019; Levin, 2011; Tan and Wright, 2018; Veiga, 2018; Weyl, 2010). A monopoly platform distorts in the spirit of Spence (1975) by internalizing only the externality new users impose on marginal users on the other side, whereas a social planner would internalize the externality imposed on average users on the other side.[1] The Spence distortion can provide a new explanation for why prices may sometimes be too high (or too low) on one or other side of a multi-sided platform. Specifically, the argument goes that after adjusting for the classical market-power distortion, a monopoly platform will set its price too high on one side compared to the planner's socially optimal price on that side when on the other side the interaction values of average users is greater than the interaction value of marginal users. This is because when setting its price, the monopoly platform does not internalize the higher interaction values of the inframarginal users on the other side.

In an earlier comment on Weyl's paper (Tan and Wright, 2018), we noted that Weyl's comparison of the monopolist's and planner's price was incorrect since it was made by directly comparing first-order conditions that should have been evaluated at different allocations. We noted the existence of other distortions that could add to or lessen the Spence distortion when comparing the two prices. In this paper, by providing a comparison of pricing outcomes directly, we characterize these additional distortions.

A fundamental additional distortion that arises when there is heterogeneity in users' interaction values in a two-sided platform setting is what we call the "displacement distortion", and it normally works in the opposite direction of the Spence distortion. It captures that the marginal user's interaction benefit on each side, that determines the externality the monopolist cares about when pricing on the other side, is itself distorted because of monopoly pricing. Thus, if the monopoly platform sets a high price on side j, the marginal user on side j must have a high interaction benefit, which means when pricing on side i, the externality the monopolist takes into account is distorted upwards compared to the interaction benefit of the marginal side j users at the socially optimal price. Other things equal, this will lead the platform to set too low a price on side i. Thus, when the interaction values of average users exceeds that of marginal users, so the Spence distortion is positive, this displacement distortion will tend to offset the Spence distortion. For instance, if the marginal user on side j at the monopoly price has an interaction benefit equal to the average users' interaction benefit at the social planner's price, then the displacement effect will exactly offset the Spence distortion on side i. In general, the net effect of the Spence and displacement distortions can therefore go in either direction.

[1] Spence (1975) finds that for a given quantity, a monopolist's choice of quality of a product reflects its impact on the value of marginal consumers rather than on the average valuation of consumers, whereas its the impact on the average valuation that matters for the socially optimal choice of quality. In a two-sided platform setting, for each side, the participation rate on that side is analogous to the quantity and the participation rate on the other side is analogous to the quality.

In the simplest case with heterogeneity only in interaction benefits, which is when the Spence distortion is at its largest, we show the displacement distortion is always in the opposite direction to the Spence distortion on at least one side of the market. Moreover, in the case interaction benefits on each side are distributed according to the generalized Pareto distribution with log-concave demand (which includes the case demands are linear), we show that the displacement distortion always exactly offsets the Spence distortion on both sides. In such a case, monopoly pricing in multi-sided platforms only involves the classical market-power distortion on each side and nothing else.

An implication of this result is that the claimed excessive merchant fees in the debit and credit card industry are not well explained by a Spence-type distortion. Monopoly debit and credit card payment platforms are typically modelled as facing cardholders and merchants that vary only in their interaction benefits.[2] Instead, excessive merchant fees for card payments have been explained in Rochet and Tirole (2002) and Wright (2012) by merchant internalization due to price coherence, by Wright (2004) due to asymmetries in pass-through rates, and in Bedre-Defolie and Calvano (2013) by an asymmetry in price discrimination possibilities due to the fact consumers decide which payment method to use for each transaction whereas merchants make an all or nothing decision whether to accept the card.

An even more extreme result is obtained when we compare a monopoly platform's prices and Ramsey prices (i.e. prices that maximize total welfare subject to the platform breaking even) in the setting with only heterogeneity in interaction benefits. In this case the Spence distortion does not arise at all, and only the classical markup and the displacement distortion remain. Because cost must be recovered in the Ramsey outcome, there is no scope to take into account the Spence distortion.

When heterogeneity is purely in membership benefits, neither the Spence distortion nor the displacement distortion arises reflecting that there is no difference between marginal and average interaction benefits. This may suggest that only a classical distortion remains. However, even though interaction benefits are constant, an additional distortion can still arise in the tariff charged per participant, which we call the scale distortion. It reflects that the profit a monopolist can extract from side j from interactions with a participant on side i depends on the number of participants on side j, which can be different in the monopolist's solution outcome compared to the social planner's solution. This means the incentive a monopoly platform has to increase the tariff on side i can be excessive or insufficient (depending upon the relative magnitude of the interaction cost and the constant interaction benefit on side j). We show that this distortion in tariffs can be as important as (or more important than) the classical market-power distortion.

We also consider the case with heterogeneity in both interactions benefits and membership benefits using Weyl's Scale-Income model. Based on our decomposition of the distortions between privately and socially optimal tariffs, all four possible distortions can arise—classical, Spence, displacement and scale. Our analysis suggests there is no particular reason why Spence distortions should be the main focus of policymakers in the case of monopoly pricing by a multi-sided platform. Indeed, we provide a numerical example to show that it is possible that the regulator does better

[2] See, for instance, Rochet and Tirole (2011) and (Wright, 2004, 2012).

reducing prices opposite the side with the smaller Spence distortion and allowing them to increase on the side opposite that with the larger Spence distortion. Thus, we question Weyl (2010)'s advice that "... the novel element in two-sided markets is that regulators should focus most on reducing price opposite a side with a large Spence distortion" (p. 1666).

The rest of the paper proceeds as follows. In Section 2 we present the model. In Section 3 we provide a general analysis allowing for both interaction and membership heterogeneity. In Section 4 we then get more specific results by applying this analysis to the case with only one dimension of heterogeneity, which is the standard setting in most models of multi-sided platforms. Section 5 concludes.

2 Model

We adopt the canonical model of a monopoly two-sided platform of Rochet and Tirole (2006). We briefly recap the model here, sticking to their notation as much as possible. There are two sides of the market: $i \in \{B, S\}$, with a continuum of potential users (with mass normalized to 1). The platform incurs fixed cost C^i per member on side i and marginal cost c per interaction between two members of the opposite sides. On each side i, users may be heterogeneous over their benefit b^i per transaction and/or their fixed benefit B^i of joining the platform. Assume $\left(B^i, b^i\right)$ is distributed according to the well behaved probability density function $f^i\left(B^i, b^i\right)$. These are drawn and known by buyers and sellers before they decide whether to join the platform. The number of transactions is then assumed to be $N^B N^S$, where N^i is the measure of users joining on side i, since each user that joins on one side is assumed to interact with all users who have joined on the other side.

We follow Weyl (2010) in allowing the platform to charge an insulating tariff to users on each side i, denoted P^i, which prescribes how much a user pays conditional on the number of participants on the other side. Note this is more general than a pure membership fee or a per-transaction fee, and ensures that the platform overcomes any unfavourable beliefs, which seems reasonable for the types of mature platforms that we have in mind. Therefore, a user on side i joins the platform if they draw $\left(B^i, b^i\right)$ such that

$$B^i + b^i N^j \geq P^i, \tag{1}$$

where $j \neq i$.

This model incorporates two special cases: (i) pure interaction heterogeneity in which B^i is the same for all users on side i, which corresponds to the situation in Rochet and Tirole (2003) (with the normalization $B^i = 0$ and $C^i = 0$) for $i \in \{B, S\}$, and (ii) pure membership heterogeneity in which b^i is the same for all users on side i for $i \in \{B, S\}$, which corresponds to the situation in Armstrong (2006).

The platform chooses an allocation (N^B, N^S), or equivalently insulating tariffs $P^B\left(N^B, N^S\right)$ and $P^S\left(N^B, N^S\right)$, to maximize its profit.

3 General analysis

Consider first the welfare created by the platform on both sides. This is

$$V\left(N^B, N^S\right) = V^B\left(N^B, N^S\right) + V^S\left(N^S, N^B\right) - C^B N^B - C^S N^S - cN^B N^S,$$

where

$$V^i\left(N^i, N^j\right) = \int_{-\infty}^{\infty}\int_{P^i(N^i,N^j)-b^iN^j}^{\infty}\left(B^i + b^i N^j\right) f^i\left(B^i, b^i\right) dB^i db^i.$$

Differentiating V with respect to N^i, and using that the derivative of V^i with respect to N^i equals P^i, the planner's tariff (that maximizes total welfare) is given by the first-order condition[3]

$$P^i = \underbrace{C^i + cN^j}_{cost} - \underbrace{\overline{b^j} N^j}_{external\ benefit\ on\ side\ j}, \tag{2}$$

where

$$\overline{b^j} = \frac{\int_{-\infty}^{\infty}\int_{P^j(N^j,N^i)-b^jN^i}^{\infty} b^j f^j\left(B^j, b^j\right) dB^j db^j}{\int_{-\infty}^{\infty}\int_{P^j(N^j,N^i)-b^jN^i}^{\infty} f^j\left(B^j, b^j\right) dB^j db^j} \tag{3}$$

is the average interaction benefit across *all* participating users on side j, evaluated at the socially optimal allocation.

The first-order condition (2) implies the planner's tariff is below cost on side i to the extent that the interaction benefits of users on side j are positive. This is standard, and reflects that when users on side i generate positive cross-side externalities to users on side j, then side i users should be subsidized to use the platform. The extent of the subsidy depends on how much an additional side i user benefits the *average* side j user.

We can contrast this with the monopoly solution. The monopoly platform obtains profit of

$$\pi\left(N^B, N^S\right) = \left(P^B\left(N^B, N^S\right) - C^B\right) N^B + \left(P^S\left(N^B, N^S\right) - C^S\right) N^S - cN^B N^S.$$

Differentiating π with respect to N^i, the monopolist's tariff is given by

$$P_m^i = \underbrace{C^i + cN_m^j}_{cost} + \underbrace{\frac{P_m^i}{\epsilon_m^i}}_{markup} - \underbrace{\widetilde{b_m^j} N_m^j}_{external\ benefit\ on\ side\ J}, \tag{4}$$

where

$$\widetilde{b_m^j} = \frac{\int_{-\infty}^{\infty} b^j f^j\left(P^j\left(N_m^j, N_m^i\right) - b^j N_m^i, b^j\right) db^j}{\int_{-\infty}^{\infty} f^j\left(P^j\left(N_m^j, N_m^i\right) - b^j N_m^i, b^j\right) db^j} \tag{5}$$

is the average interaction benefit across *marginal* users on side j evaluated at the monopoly allo-

[3]The formal working is given in the Appendix. Like Rochet and Tirole (2006) and Weyl (2010), throughout the paper we assume that the first-order condition characterizes the outcome that maximizes the corresponding objective function.

cation, which we denote by a subscript m, and

$$\epsilon^i_m = -\frac{\partial N^i}{\partial P^i}\frac{P^i_m}{N^i_m}$$

is the price elasticity of demand on side i evaluated at the monopoly allocation (thus, here $\frac{\partial N^i}{\partial P^i}$ is also evaluated at the monopoly allocation). Note we denote the classical market-power term as "markup" for short. As can be seen from (4), the extent to which the platform optimally reduces its tariff to side i users to take into account positive externalities depends on the benefit an additional side i user creates for the *marginal* side j user.

Comparing the two first-order conditions and taking into account the different outcomes, we can write the monopoly tariff as

$$P^i_m = \underbrace{C^i + cN^j - \overline{b^j}N^j}_{\textit{socially optimal price}} + \underbrace{\frac{P^i_m}{\epsilon^i_m}}_{\textit{markup}} + \underbrace{\left(\overline{b^j} - \widetilde{b^j}\right)N^j}_{\textit{Spence distortion}} + \underbrace{\left(\widetilde{b^j} - \widetilde{b^j_m}\right)N^j}_{\textit{displacement distortion}} + \underbrace{\left(\widetilde{b^j_m} - c\right)\left(N^j - N^j_m\right)}_{\textit{scale distortion}}, \tag{6}$$

where the outcome without a subscript represents the planner's allocation. This result says that there are four ways in which the monopolist's tariff charged to users differs from the planner's. Apart from the classical markup and Spence distortions emphasized by the existing literature, two additional distortions arise—a displacement distortion and a scale distortion.

The Spence distortion reflects that the monopoly platform internalizes only network externalities to marginal users rather than all participating users. So the monopoly platform will set its tariff too high on one side compared to the planner's tariff on that side when on the other side of the platform the interaction values of average users is greater than the interaction value of marginal users. This is because when setting its tariff, the monopoly platform does not internalize the higher interaction values of the inframarginal users on the other side. Note that the Spence distortion captures the different incentives of the monopoly platform and the planner in setting the tariff on side i taking as given the number of users on side j, which we fix at the planner's allocation.

The displacement distortion reflects that the interaction benefits of marginal users on the opposite side will generally differ when comparing the privately optimal outcome with the socially optimal outcome. This, in turn, reflects that monopoly tariffs will generally differ from efficient tariffs. To the extent that the monopoly tariff exceeds the efficient tariff on side j, the interaction benefit of the marginal user on side j will be higher for the monopoly outcome, and so the displacement distortion defined in (6) will be negative. This makes sense. The monopoly two-sided platform considers the effect of its pricing on side i on the marginal user's interaction benefit on side j since this determines how much the monopolist can extract on side j. If the marginal user's interaction benefit on side j is distorted upwards due to monopoly pricing on side j, then the platform has a reason to lower its tariff on side i so as to extract this high profit on side j, thereby reducing its incentive to set higher tariffs on side i.

The last distortion noted in (6) is the scale distortion. It reflects that even after controlling for the Spence and displacement distortions, so that a monopoly platform's focus on the marginal user's interaction benefit (or tariff) on the other side is appropriate, there is still a distortion since the number of participants on the other side is different in the monopoly outcome compared to the socially optimal outcome. This reflects that the profit (or surplus) a monopolist (or planner) can extract on side j from the users' interactions with an additional side i participant depends on the number of participants on side j, which is generally different in the monopoly outcome compared to the planner's outcome. In particular, the monopolist considers surplus (or profit) $\widetilde{b_m^j} - c$ over N_m^j participants on side j, whereas after controlling for the Spence and displacement distortions, the planner measures the same surplus $\widetilde{b_m^j} - c$ over N^j participants instead. Thus, the scale distortion arises because there may be a different number of users on the other side in the monopoly's outcome compared to the planner's outcome. It will be positive on side i if the surplus created by interaction benefits for the marginal users on side j is positive at the monopoly tariffs, and if there is more participation in the planner's outcome on side j than the monopolist's outcome, both of which represent the standard case.

In our view, the decomposition between the monopoly tariff and the planner's tariff in (6) is the most natural way to compare the two tariffs. It involves defining the Spence distortion taking the number of users on side j as fixed at the planner's allocation. We could have instead defined the Spence distortion fixing the number of users on side j at the monopoly allocation. The decomposition then becomes

$$P_m^i = \underbrace{C^i + cN^j - \overline{b^j}N^j}_{\textit{socially optimal price}} + \underbrace{\frac{P_m^i}{\epsilon_m^i}}_{\textit{markup}} + \underbrace{\left(\overline{b_m^j} - \widetilde{b_m^j}\right) N_m^j}_{\textit{Spence distortion}} + \underbrace{\left(\overline{b^j} - \overline{b_m^j}\right) N^j}_{\textit{displacement distortion}} + \underbrace{\left(\overline{b_m^j} - c\right)\left(N^j - N_m^j\right)}_{\textit{scale distortion}},$$

so this requires evaluating the displacement distortion and scale distortion with respect to the average interaction benefits rather than with respect to the marginal interaction benefits. Although the interpretations are similar, we will stick to measuring the Spence distortion at the planner's allocation throughout the rest of the paper, which makes the remaining distortions easier to interpret.

In (6) we have characterized the distortion in the tariff paid by each user. In some cases, the researcher may be more interested in the distortion of the per-interaction (or per-transaction) price. The per-transaction price has been the focus of attention in the two-sided market settings in which there is pure interaction heterogeneity (e.g. Rochet and Tirole (2003)), reflecting that the insulating tariff then is simply a constant per-transaction price paid on each interaction. With this in mind, let $p^i \equiv P^i/N^j$ be the price charged to users on side i for each interaction with the N^j participants on side j. Then dividing through by N^j, (2) becomes

$$p^i = \underbrace{\frac{C^i}{N^j} + c}_{\textit{cost}} - \underbrace{\overline{b^j}}_{\textit{external benefit on side } j} \tag{7}$$

while dividing through by N_m^j, (4) becomes

$$p_m^i = \underbrace{\frac{C^i}{N_m^j} + c}_{cost} + \underbrace{\frac{p_m^i}{\epsilon_m^i}}_{markup} - \underbrace{\widetilde{b_m^j}}_{external\ benefit\ on\ side\ j}. \tag{8}$$

Note the definition of ϵ_m^i is unchanged since $-\frac{\partial N^i}{\partial P^i}\frac{P_m^i}{N_m^i} = -\frac{\partial N^i}{\partial p^i}\frac{1}{N_m^j}\frac{p_m^i N_m^j}{N_m^i} = -\frac{\partial N^i}{\partial p^i}\frac{p_m^i}{N_m^i}$, where all derivatives are evaluated at the monopoly outcome. Comparing (7) with (8), the monopoly price can be written as

$$p_m^i = \underbrace{\frac{C^i}{N^j} + c - \overline{b^j}}_{socially\ optimal\ price} + \underbrace{\frac{p_m^i}{\epsilon_m^i}}_{markup} + \underbrace{\overline{b^j} - \widetilde{b^j}}_{Spence\ distortion} + \underbrace{\widetilde{b^j} - \widetilde{b_m^j}}_{displacement\ distortion} + \underbrace{\frac{C^i}{N_m^j} - \frac{C^i}{N^j}}_{cost\ distortion}. \tag{9}$$

By writing prices on a per-interaction basis, we can ignore the scale distortion that arises when comparing the per-user tariff across the monopolist's and planner's allocations. However, doing so introduces a cost distortion to the extent that there are costs arising per member which will be allocated over a different number of interactions in the two different solutions (i.e. when $N_m^j \neq N^j$). Taking into account that the monopolist's choice of output is generally lower than the planner's output, this cost distortion is generally positive, and a reason why the monopolist's per-interaction price will be higher than that of the planner.

4 Restricting to one dimension of heterogeneity

To further interpret the general characterization of distortions developed in Section 3, we focus on some special cases in this section. Specifically, we consider the case with just interaction heterogeneity (Section 4.1), just membership heterogeneity (Section 4.2), and a case in which the two types of heterogeneity are jointly determined by a single dimension of heterogeneity (Section 4.3).

4.1 Pure interaction heterogeneity

Rochet and Tirole (2003) consider the case with pure interaction heterogeneity. Weyl notes (p.1652) that in this case, the Spence distortion is exactly equal to the per-interaction surplus, which is necessarily positive. We confirm the Spence distortion, properly measured, is indeed the per-interaction surplus, but show that in this setting, it can be completely offset (or in some cases more than offset) by other distortions.

Without membership benefits or costs to consider, users on side i make use of the platform if $b^i \geq p^i$, which follows from (1). Let $f^i\left(b^i\right)$ be the density function for b^i. Then the number of participating users on side i is given by $N^i = \int_{p^i}^{\infty} f^i\left(b^i\right) db^i$. Then (3) simplifies to

$$\overline{b^j} = \frac{\int_{p^j}^{\infty} b^j f^j\left(b^j\right) db^j}{\int_{p^j}^{\infty} f^j\left(b^j\right) db^j},$$

while (5) can simply be written as $\widetilde{b_m^j} = p_m^j$, since by construction, the interaction benefit of the marginal user on side j just equals the per-interaction price on side j. Apart from these simplifications, the only other change in (9) is that the cost distortion disappears given $C^i = 0$.

With no membership benefits or costs, it is natural to focus on distortions in the per-interaction price. Thus, we get from (9) that

$$p_m^i = \underbrace{c - \overline{b^j}}_{\textit{socially optimal price}} + \underbrace{\frac{p_m^i}{\epsilon_m^i}}_{\textit{markup}} + \underbrace{\overline{b^j} - \widetilde{b^j}}_{\textit{Spence distortion}} + \underbrace{\widetilde{b^j} - \widetilde{b_m^j}}_{\textit{displacement distortion}}. \tag{10}$$

Given at least one of the monopoly prices must be higher than the efficient price, we are able to obtain the following result.

Proposition 1. With only interaction benefits and focusing on prices, the Spence distortion is always positive on both sides, while the displacement distortion is negative on at least one side.

Proof. First note that $\overline{b^j} - \widetilde{b^j} > 0$ follows from the definition of $\overline{b^j}$ and that $\widetilde{b^j} = p^j$. Thus, the Spence distortion is always positive. Given $\widetilde{b^j} = p^j$ and $\widetilde{b_m^j} = p_m^j$, (10) with $i = B$ implies

$$p_m^B + p_m^S = p^B + p^S + \frac{p_m^B}{\epsilon_m^B} + \overline{b^S} - \widetilde{b^S}.$$

Since the classical market-power and Spence distortions are positive, $p_m^B + p_m^S > p^B + p^S$. Therefore, it must be that $\widetilde{b^B} - \widetilde{b_m^B} < 0$ and/or $\widetilde{b^S} - \widetilde{b_m^S} < 0$. □

To obtain more specific results, we consider the case in which interaction benefits are distributed according to the generalized Pareto distribution. Specifically, assume $f^i(b) = \lambda^i \left(1 - \lambda^i \sigma^i \left(b - \mu^i\right)\right)^{\frac{1-\sigma^i}{\sigma^i}}$ for $i = B, S$ with $b \in \left[\mu^i, \mu^i + \frac{1}{\lambda^i \sigma^i}\right]$, where μ^i is a shift parameter, λ_i is a scale parameter and σ_i is a shape parameter. The demand on side i is then $N^i(p) = \left(1 - \lambda^i \sigma^i \left(p - \mu^i\right)\right)^{\frac{1}{\sigma^i}}$ provided $\mu^i \leq p^i \leq \mu^i + \frac{1}{\lambda^i \sigma^i}$. We assume $\sigma^i > 0$ so that demand is log concave on side i. Note that $\sigma^i = 1$ captures the case with linear demand on side i.

Noting that $\overline{b^j} = \frac{1 + \lambda^j \left(p_S^j + \mu^j \sigma^j\right)}{\lambda^j (1 + \sigma^j)}$ and solving (7) for the planner's prices, we find

$$p^i = \frac{\lambda^j - \lambda^i \left(1 + \sigma^i\right) \left(1 - c\lambda^j \sigma^j\right) - \lambda^i \lambda^j \left(\mu^j \left(1 + \sigma^i\right) \sigma^j - \mu^i \sigma^i\right)}{\lambda^i \lambda^j \left(\sigma^i + \sigma^j + \sigma^i \sigma^j\right)}$$

for $i = B, S$. We can compare this to the monopolist's price, which is determined by solving (8) for $i = B, S$. The solution can be written as

$$p_m^i = p^i + m,$$

where

$$m = \frac{p_m^i}{\epsilon_m^i} = \frac{p_m^j}{\epsilon_m^j} = \frac{\lambda^i \sigma^i + \lambda^j \sigma^j + \lambda^i \lambda^j \sigma^i \sigma^j \left(\mu^i + \mu^j - c\right)}{\lambda^i \lambda^j \left(\sigma^i + \sigma^j + \sigma^i \sigma^j\right)}$$

is the symmetric markup that applies to both sides in the monopolist's solution. Then based on the decomposition in (9), we find the Spence distortion is also equal m, and the displacement distortion equals $-m$, so there is perfect offset. The only difference between the monopolist's and planner's prices is the classical markup term m. We state this result as a proposition.

Proposition 2. With only interaction benefits, the Spence distortion is always exactly offset by the displacement distortion when the usage benefits on each side are distributed according to the generalized Pareto distribution with log-concave demand.

Proposition 2 is surprising since the case in which there is only heterogeneity in interaction benefits represents the case in which the Spence distortion is at its maximum. Indeed, under the generalized Pareto distribution, the Spence distortion is the same magnitude as the classical market-power distortion. Based on the existing literature that emphasized only the classic and Spence distortions of monopoly pricing, one might conclude that for this class of demands, monopoly prices involve double the normal mark-up due to market-power. Instead, as shown in Proposition 2, taking into account the displacement distortion, there is in fact no additional distortion in the prices on each side beyond the usual markup due to market power.

One may wonder if the result in Proposition 2 in which the Spence distortion and displacement distortion exactly offset holds more generally. The following example based on the power distribution shows that in general the two distortions do not exactly offset. In the example, $f^i(b) = \lambda^i b^{\lambda^i - 1}$ with $b \in [0,1]$, so the demand function on side i is given by $N^i(p) = 1 - p^{\lambda^i}$. Let $\lambda^B = 1/3$, $\lambda^S = 2/3$, and $c = 1$. Using straightforward calculations, the decomposition in (10) is different on each side, and becomes

$$\underbrace{0.639}_{\textit{privately optimal } p^B} = \underbrace{0.328}_{\textit{socially optimal } p^B} + \underbrace{0.308}_{\textit{markup}} + \underbrace{0.295}_{\textit{Spence distortion}} - \underbrace{0.294}_{\textit{displacement distortion}}$$

for side B and

$$\underbrace{0.669}_{\textit{privately optimal } p^S} = \underbrace{0.377}_{\textit{socially optimal } p^S} + \underbrace{0.308}_{\textit{markup}} + \underbrace{0.295}_{\textit{Spence distortion}} - \underbrace{0.311}_{\textit{displacement distortion}}$$

for side S. For this example, the displacement distortion does not quite fully offset the Spence distortion on side B, but more than offsets the Spence distortion on side S.

If we instead consider the distortions arising in the tariff each user pays (i.e. the total fee a user pays for all her interactions), from the comparison in (6) we can see that the additional scale distortion remains. This reflects that while monopoly pricing can be explained purely by the classical markup on top of the planner's prices, this can lead to an additional positive or negative distortion for users' total fee depending on whether the monopoly price on the opposite side exceeds marginal cost and whether there is more or less participation on the other side at the planner's allocation compared to the monopolist's allocation. Since the marginal cost is incurred per interaction, but the platform sets a price on each side of the interaction, it is quite possible that $\widetilde{b^j_m} - c$ or equivalently $p^j_m - c$, is negative on one or both sides. As a result, the scale distortion can be positive, negative, or zero, even when assuming symmetric demand.

For the tariff per user, the comparisons based on the power distribution with $\lambda^B = 1/3$, $\lambda^S = 2/3$, and $c = 1$ implies

$$\underbrace{0.150}_{\textit{privately optimal } P^B} = \underbrace{0.157}_{\textit{socially optimal } P^B} + \underbrace{0.073}_{\textit{markup}} + \underbrace{0.141}_{\textit{Spence distortion}} - \underbrace{0.140}_{\textit{displacement distortion}} - \underbrace{0.081}_{\textit{scale distortion}}$$

for side B and

$$\underbrace{0.093}_{\textit{privately optimal } P^S} = \underbrace{0.117}_{\textit{socially optimal } P^S} + \underbrace{0.043}_{\textit{markup}} + \underbrace{0.092}_{\textit{Spence distortion}} - \underbrace{0.097}_{\textit{displacement distortion}} - \underbrace{0.062}_{\textit{scale distortion}}$$

for side S. In this example, the displacement and scale distortions together more than offset the positive classical and Spence distortion on each of the sides, so that the monopolist's tariffs per user are actually lower than the planner's on both sides. If instead, $\lambda^B = 1/2$, $\lambda^S = 1/2$, and $c = 4/9$, then the scale distortion is exactly zero, although in this case the displacement distortion more than offsets the Spence distortion. Finally, when $\lambda^B = 1/2$, $\lambda^S = 1/2$, and $c = 4/10$, the scale distortion is positive, but the sum of the Spence, displacement and scale distortions still remains negative.

The case with pure interaction heterogeneity also affords a simple comparison between the monopoly prices and the Ramsey prices (i.e. the prices that maximize total welfare subject to the platform being able to cover its costs). First note that the platform's breakeven constraint must be binding when maximizing welfare. To see this, recall for the unconstrained welfare maximization problem we have $p^i = c - \overline{b^j}$, so $p^B + p^S - c = c - \overline{b^B} - \overline{b^S}$. Suppose $p^B + p^S \geq c$. This implies $\widetilde{b^B} + \widetilde{b^S} \geq c$, which implies $\overline{b^B} + \overline{b^S} > c$ since $\overline{b^i} > \widetilde{b^i} = p^i$ for the case with pure interaction heterogeneity. But that also implies $p^B + p^S - c = c - \overline{b^B} - \overline{b^S} < 0$, a contradiction. Thus, it must be that $p^B + p^S < c$, and the Ramsey solution satisfies $p_r^i = c - p_r^j = c - \widetilde{b_r^j}$, where $\widetilde{b_r^j} = p_r^j$ is the marginal user's interaction value. Comparing this solution to the standard monopoly solution as defined by $p_m^i = c + \frac{p_m^i}{\epsilon_m^i} - c - \widetilde{b_m^j}$, we have that

$$p_m^i = \underbrace{c - \widetilde{b_r^j}}_{\textit{Ramsey price}} + \underbrace{\frac{p_m^i}{\epsilon_m^i}}_{\textit{markup}} + \underbrace{\widetilde{b_r^j} - \widetilde{b_m^j}}_{\textit{displacement distortion}}. \tag{11}$$

We summarize the result in Proposition 3.

Proposition 3. With only interaction benefits, the monopoly price on each side only differs from the Ramsey optimal price by a classical markup and a displacement distortion. There is no Spence distortion.

The result implies that the Spence distortion is not relevant for understanding how the monopoly platform's prices differ from Ramsey prices in a setting with pure interaction heterogeneity. Given unconstrained welfare maximization requires the platform make a loss (i.e. the total price level is less than total costs), reflecting the positive externalities it needs to get users on each side to internalize, Ramsey pricing involves the platform just breaking even. This means the Ramsey price on one side is pinned down by cost less the interaction benefits of the marginal user on the other side, and the interaction benefits of the average user do not play any role.

4.2 Pure membership heterogeneity

Armstrong (2006) considers the case with pure membership heterogeneity. In this case, b^i is constant across all users on side $i = B, S$, so the Spence distortion and displacement distortion disappear. Given b^i is constant, users on side i make use of the platform if $B^i + b^i N^j \geq P^i$, which follows from (1). Let $f^i\left(B^i\right)$ be the density function for B^i. Then the number of participating users on side i is given by $N^i = \int_{P^i - b^i N^j}^{\infty} f^i\left(B^i\right) dB^i$. The comparison in (6) becomes

$$P_m^i = \underbrace{C^i + cN^j - b^j N^j}_{\textit{socially optimal price}} + \underbrace{\frac{P_m^i}{\epsilon_m^i}}_{\textit{markup}} + \underbrace{\left(b^j - c\right)\left(N^j - N_m^j\right)}_{\textit{scale distortion}}. \tag{12}$$

When considering the difference between the monopolist's and planner's choices of the membership fees, only the classical market-power and scale distortions remain. As noted previously, the scale distortion can be positive (reinforcing the usual classical distortion) or negative (reducing the classical distortion).

To illustrate, consider a setting in which $f^i(B^i) = \frac{1}{\lambda}$ for $i = B, S$ over $[\underline{B}, \overline{B}]$, so $\overline{B} - \underline{B} = \lambda$. Assume symmetric costs, so $C^B = C^S = C < \overline{B}$. Define the surplus created by an interaction as $\Delta = b^B + b^S - c$. Assume $0 < \Delta < C - \underline{B} < \lambda$, which ensures that the total interaction benefit to both sides exceeds the cost of an interaction, second-order conditions hold for the monopolist's and planner's problem, and not all consumers participate at the monopolist's and planner's solution. Then the scale distortion on side i becomes

$$\left(\frac{b^j - c}{\lambda - \Delta}\right) m, \tag{13}$$

where $m = \frac{\lambda(\overline{B} - C)}{2\lambda - \Delta} > 0$ is the classical market-power distortion in (4). Clearly, (13) can be positive, zero, or negative on side i, depending on whether b^j is greater than, equal to, or less than c. Note in particular, if $b^i = \lambda$ (which implies $b^j < c$ given $\Delta < \lambda$) then the scale distortion equals $-m$, so the monopoly platform sets its membership fees at the socially efficient level since the scale distortion completely offsets the classical market-power distortion. On the other hand, if $b^i = b^j = \frac{\lambda}{3} + \frac{2c}{3}$, then the scale distortion equals m, so the overall markup in membership fees on each side are double the usual classical markup.

4.3 Scale-income model

In general, users on each side i differ in both their membership benefit B^i and their usage benefit b^i. The general analysis of Section 3 then applies. To say something more concrete for this general case, we adopt Weyl's preferred Scale-Income (SI) model (see Section V of Weyl (2010)). Specifically, in the SI model, $b^i = \beta^i B^i$, where β^i is a constant on side i. Thus, a high income user that obtains a high value from platform membership also puts a high (low) value on interacting on the platform if $\beta > 0$ (< 0). In this way, users still vary in how much they value both membership and usage, but with just one dimension of heterogeneity.

We use the SI model to consider the claim of Weyl, that (p.1666) "the novel element in two-sided markets is that regulators should focus most on reducing price opposite a side with a large Spence distortion. Thus regulators of ISPs should focus on limiting prices to Web sites (net neutrality) if there is more (interaction) surplus among loyal users than among highly profitable Web sites. But if the situation is reversed, forcing ISPs to reduce prices and build more line to consumer homes may be a higher priority."

As we have seen in earlier sections, a large Spence distortion on one side may be offset by a negative displacement and/or scale distortion on that side. With pure interaction heterogeneity, the Spence distortion was generally offset by displacement distortion in the opposite direction. With pure membership heterogeneity, by definition, there is no Spence distortion. This suggests that even when we combine the two types of heterogeneity, Spence distortions will not necessarily drive pricing distortions. As such, our analysis suggests there is no reason in general why a regulator will want to lower prices on the side opposite that which generates the largest Spence distortion.

We illustrate with a numerical example motivated by the regulation of ISPs, in which B represents users and S represents websites. Assume $\beta^B > 0$ (high-income consumers value participation and interactions more) and $\beta^S < 0$ (high-value websites value interactions more but face higher fixed costs of participating). We consider parameter values such that the Spence distortion produced by websites is larger than that produced by users which according to Weyl's claim, implies regulators of ISPs should focus on limiting prices to users rather than Web sites.

In particular, if $f^B(b^B) = \frac{1}{7.5}\left(1 - \frac{b^B}{5}\right)^{-\frac{1}{3}}$ with $b^B \in [0,5]$, $f^S(b^S) = \frac{1}{3}\left(1 - \frac{b^S}{6}\right)$ with $b^S \in [0,6]$, $\beta^B = 1$, $\beta^S = -6$, $C^B = 3$, $C^S = 0.5$ and $c = 3.5$, then

$$\underbrace{4.847}_{\textit{privately optimal } P^B} = \underbrace{3.998}_{\textit{socially optimal } P^B} + \underbrace{1.796}_{\textit{markup}} + \underbrace{1.557}_{\textit{Spence distortion}} - \underbrace{2.350}_{\textit{displacement distortion}} - \underbrace{0.154}_{\textit{scale distortion}}$$

for side B and

$$\underbrace{0.564}_{\textit{privately optimal } P^S} = \underbrace{0.249}_{\textit{socially optimal } P^S} + \underbrace{0.237}_{\textit{markup}} + \underbrace{1.165}_{\textit{Spence distortion}} - \underbrace{1.263}_{\textit{displacement distortion}} + \underbrace{0.176}_{\textit{scale distortion}}$$

for side S. Note the Spence distortion $\left(\overline{b^S} - \widetilde{b^S}\right) N^S$ generated from side S onto side B's pricing (1.557) is larger than that generated from side B onto side S's pricing (1.165), but the total distortion generated from side B onto side S's pricing (0.078) is larger than that generated from side S onto side B's pricing (-0.948).[4]

Consistent with the logic based on regulating the side opposite that with greater total distortions rather than just the Spence distortion, we find that the welfare maximizing tariffs subject to the

[4] Here, by total distortion, we refer to the sum of the Spence, displacement and scale distortions.

platform covering its costs (i.e. Ramsey optimal tariffs) involve regulating the tariff on side S down from 0.564 to 0.537 while allowing the tariff on side B to increase from 4.847 to 4.892 in order to cover costs. Thus, while focusing on the Spence distortions alone may suggest regulating the tariffs charged to users (since the Spence distortion generated from websites is higher), in this example the Ramsey optimal tariffs actually involve regulating the tariff charged to the side of websites and allowing the tariff charged to users to increase. This result holds despite the fact that the example was constructed so that the classical markup is much larger on the user side, which might suggest regulation on the user side should dominate. This illustrates the potential importance for regulatory policy of accounting for more than just the classical markup and Spence distortion in multi-sided platforms.

5 Conclusion

In this short paper we have provided a general treatment of the monopoly pricing distortions that arise on platforms. One of the key normative findings of the earlier literature was the tendency of a monopoly platform to focus too much on the surplus of marginal rather than average users (a so-called Spence distortion). Indeed Weyl (2010) notes (p.1652) “This Spence distortion is likely more important in two-sided markets than the contexts for which it was originally conceived.” We showed there are two other important distortions that arise in two-sided platform settings with cross-group network effects, a displacement distortion and a scale distortion.

We find for reasonable cases, these additional distortions can completely offset the Spence distortion so that only the classical market-power distortion remains, create positive or negative distortions aside from the classical ones in settings where the Spence distortion does not exist, or reverse the pattern of distortions implied by focusing only on the Spence distortion, thereby potentially reversing policy implications focused only on the Spence distortion.

Ultimately, the direction and size of the different distortions we have documented for multisided platforms is an empirical matter. Structural estimation of a platform’s demand functions and costs could be one way to quantify them.

6 Appendix

We want to show that $\frac{\partial V^i}{\partial N^i} = P^i$ and $\frac{\partial V^j}{\partial N^i} = \overline{b^j} N^j$. First note that

$$\frac{\partial V^i}{\partial N^i} = -\frac{\partial P^i\left(N^i, N^j\right)}{\partial N^i} P^i\left(N^i, N^j\right) \int_{-\infty}^{\infty} f^i\left(P^i\left(N^i, N^j\right) - b^i N^j, b^i\right) db^i$$

and

$$N^i = \int_{-\infty}^{\infty} \int_{P^i(N^i,N^j)-b^i N^j}^{\infty} f^i\left(B^i, b^i\right) dB^i db^i$$

so

$$\frac{\partial N^i}{\partial P^i} = -\int_{-\infty}^{\infty} f^i\left(P^i\left(N^i, N^j\right) - b^i N^j, b^i\right) db^i.$$

Using that $\frac{\partial P^i\left(N^i,N^j\right)}{\partial N^i}=-\frac{1}{\frac{\partial N^i}{\partial P^i}}$ we get that $\frac{\partial V^i}{\partial N^i}=P^i\left(N^i,N^j\right)$.

Next consider $\frac{\partial V^j}{\partial N^i}$. Then

$$V^j\left(N^j,N^i\right)=\int_{-\infty}^{\infty}\int_{P^j(N^j,N^i)-b^jN^i}^{\infty}\left(B^j+b^jN^i\right)f^j\left(B^j,b^j\right)dB^jdb^j.$$

So

$$\frac{\partial V^j}{\partial N^i}=\int_{-\infty}^{\infty}\int_{P^j(N^j,N^i)-b^jN^i}^{\infty}b^jf^j\left(B^j,b^j\right)dB^jdb^j-P^j\left(N^j,N^i\right)\int_{-\infty}^{\infty}\left(\frac{\partial P^j}{\partial N^i}-b^j\right)f^j\left(P^j\left(N^j,N^i\right)-b^jN^i,b^j\right)db^j. \tag{14}$$

Since the marginal consumer on side j for any b^j must have B^j that satisfies $B^j+b^jN^i=P^j\left(N^j,N^i\right)$, it must be that the second term in (14) is zero, and

$$\frac{\partial V^j}{\partial N^i}=\int_{-\infty}^{\infty}\int_{P^j(N^j,N^i)-b^jN^i}^{\infty}b^jf^j\left(B^j,b^j\right)dB^jdb^j=\overline{b^j}N^j.$$

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